

Lowest Common Multiple

“The smallest common multiple of two or more numbers is called the lowest common multiple (LCM)”

E.g. Multiples of 8 are 8, 16, 24, 32 ...
 Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 ...
 The LCM of 3 and 8 is 24.

“To find the lowest common multiple (LCM) of two or more numbers, list the multiples of the larger number and stop when you find a multiple of the other

Example: Find the lowest common multiple of 6 and 9.
 Solution: List the multiples of 9 and stop when you find a multiple of 6.
 Multiples of 9 are 9, 18 ...
 Multiples of 6 are 6, 12, 18 ...
 The LCM of 6 and 9 is 18

The least common multiple, or LCM, is another number that's useful in solving many math problems.

Let's find the LCM of 30 and 45. One way to find the least common multiple of two numbers is to first list the prime factors of each number.

$$30 = 2 \times 3 \times 5$$

$$45 = 3 \times 3 \times 5$$

Then multiply each factor the greatest number of times it occurs in either number. If the same factor occurs more than once in both numbers, you multiply the factor the greatest number of times it occurs.

2: one occurrence

3: two occurrences

5: one occurrence

$$2 \times 3 \times 3 \times 5 = 90 \rightarrow \text{LCM}$$

After you've calculated a least common multiple, always check to be sure your answer can be divided evenly by both numbers.

Factor

“A factor of a given number is every number that divides exactly into that number”

Example: Write down all factors of 10.

$10 = 2 \times 5$, so numbers 2 and 5 are factors of 10.

Also $10 = 10 \times 1$, so 10 and 1 are factors of 10.

The factors of 10 are 1, 2, 5, 10.

NOTE: Number 1 and the number itself are always factors of any number.

Prime and composite numbers

“A prime number has exactly 2 factors, the number itself and 1”

In other words, the prime number can be divided only by 1 and by itself.

NOTE: 0 and 1 are not prime numbers.

Example: 5 is a prime number, because the only factors it has are 1 and 5.

The prime numbers less than 20 are 2,3,5,7,11,13,17,19

Example: Find all prime factors of 30.

Solution: All the factors of 30 are 30, 15, 10, 6, 5, 3, 2, 1

But only 5, 3 and 2 are prime numbers. Therefore all prime factors of 30 are 2, 3 and 5.

“A composite number has at least one more factor than the number itself or 1”

In fact, all whole numbers that are not prime are composite except for 1 and 0, which are not prime and not composite.

The composite numbers less than 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

Divisibility rules

The simple divisibility rules will help you to find factors of a number.

The number is divisible by:

- 2 if the last digit is 0, 2, 4, 6, or 8 (example: 12346)
- 3 if the sum of digits in the number are divisible by 3 (example: 1236, because $1+2+3+6 = 12 = 3 \times 4$)
- 4 if the last 2 digits are divisible by 4 (example: 897544, because $44 = 4 \times 11$)

- 5 if the last digit is 0 or 5 (example: 178965 or 40980)
- 6 if it is divisible by 2 and 3
- 7 sorry, no rule (you have to divide)
- 8 if the last 3 digits are divisible by 8 (example: 12487080, because $080 = 8 \times 0$)
- 9 if the sum of digits is divisible by 9 (example: 234612, because $2+3+4+6+1+2 = 18 = 9 \times 2$)
- 10 if the last digit is 0 (example: 99990)
- 100 if the last 2 digits are 0 (example 987600)

NOTE: If a number is divisible by 2 factors, it is also divisible by the product of these factors.

Example 1: Number 18 is divisible by 2 and 3, so it must be divisible by $2 \times 3 = 6$.

Example 2: Number 945 is divisible by 9 (why?) and by 5 (why?), so it must be divisible by $9 \times 5 = 45$. (Can you check it?)

Common factors

“When two (or more) numbers have the same factor, that factor is called a common factor”

Example: Find all the common factors of 12 and 18.

Factors of 12 are 1, 2, 3, 4, 6, 12.

Factors of 18 are 1, 2, 3, 6, 18.

The common factors of 12 and 18 are 1, 2, 3 and 6.

Highest Common Factor (H.C.F)

“The Highest Common Factor (H.C.F) of two (or more) numbers is the largest number that divides evenly into both numbers”

In other words the H.C.F is the largest of all the common factors.

The common factors of 12 and 18 are 1, 2, 3 and 6.

The largest common factor is 6, so this is the H.C.F. of 12 and 18.

It is very easy to find a H.C.F. of small numbers, like 6 and 9 (it is 3) or 8 and 4 (it is 4).

The best way is to keep finding the factors of the smaller number, starting from the largest factor. The first factor of the smaller number that is also a factor of the larger number is a H.C.F.

The H.C.F is useful when simplifying fractions.

Finding the H.C.F. of big numbers

For larger numbers you can use the following method:

1. Find all prime factors of both numbers.
2. Write both numbers as a multiplication of prime numbers.
3. Find which factors are repeating in both numbers and multiply them to get H.C.F

Example: Find the Highest Common Factor (H.C.F.) of 240 and 924.

Solution: Finding all prime factors of 240. We will start with the smallest prime number and we will divide 240 into it if we can (the divisibility rules come handy). We will do the same with the result (or quotient), and we will keep dividing by prime numbers until we have 1 as a quotient. Each time we write the prime factor to the right and the quotient below

Finding all prime factors of 240:

240	2	2 is a factor of 240; 240 divided by 2 is 120
120	2	2 is a factor of 120; 120 divided by 2 is 60
60	2	2 is a factor of 60; 60 divided by 2 is 30
30	2	2 is a factor of 30; 30 divided by 2 is 15
15	3	3 is a factor of 15; 15 divided by 3 is 5
5	5	5 is a factor of 5; 5 divided by 5 is 1
1		

240 = 2 x 2 x 2 x 2 x 3 x 5

Finding all prime factors of 924:

924	2	2 is a factor of 924; 924 divided by 2 is 462
462	2	2 is a factor of 462; 462 divided by 2 is 231
231	3	3 is a factor of 231; 213 divided by 3 is 77
77	7	7 is a factor of 77; 77 divided by 7 is 11
11	11	11 is a factor of 11; 11 divided by 11 is 1
1		

924 = 2 x 2 x 3 x 7 x 11

Multiply the factors which repeat in both numbers to get the H.C.F.

The Highest Common Factor is $2 \times 2 \times 3 = 12$

Multiples Common Multiples

“When you multiply a given whole number by any other whole number, the result is a multiple of that number”

For example, 5 is the first multiple of 5 (because $5 \times 1 = 5$), 10 is the second multiple of 5, and so on.

Example: Write down the first 3 multiples of 8.

Solution: $8 \times 1 = 8$, $8 \times 2 = 16$, $8 \times 3 = 24$, so the first 3 multiples of 8 are 8, 16, 24.

Example: Write down all multiples of 3 greater than 10 but smaller than 20.

Solution: Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24,...

The desired ones are 12, 15, 18.

The common multiples of two numbers are multiples of both numbers.

Lowest common multiple (L.C.M.)

“The Lowest Common Multiple (L.C.M) is the smallest number that is a common multiple of two or more numbers”

For example, the L.C.M of 3 and 5 is 15.

The simple method of finding the L.C.M of smaller numbers is to write down the multiples of the larger number until one of them is also a multiple of the smaller number.

Example: Find the Lowest Common Multiple of 8 and 12.

Solution: Multiples of 12 are 12, 24...

24 is also a multiple of 8, so the L.C.M of 8

Finding L.C.M. of big numbers

1. Find all the prime factors of both numbers
2. Multiply all the prime factors of the larger number by those prime factors of the smaller number that are not already included

Example: Find the Lowest Common Multiple (L.C.M.) of 240 and 924.

From the example of finding the H.C.F. we know the prime factors of both numbers.

$$924 = 2 \times 2 \times 3 \times 7 \times 11$$

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$

$$\text{The L.C.M. is } 2 \times 2 \times 3 \times 7 \times 11 \times 2 \times 2 \times 5 = 924 \times 20 = 18.480$$