ANGLES

An angle is the figure formed by two rays sharing a common endpoint, called the vertex of the angle. The magnitude of the angle is the "amount of rotation" that separates the two rays, and can be measured by considering the length of circular arc swept out when one ray is rotated about the vertex to coincide with the other.

The word *angle* comes from the Latin word *angulus*, meaning "a corner". The word *angulus* is a diminutive, of which the primitive form, *angus*, does not occur in Latin. Cognate words are the Latin *angere*, meaning "to compress into a bend" or "to strangle", the Greek ἀγκύλος *(ankylos)*, meaning "crooked, curved," and the English word "ankle."

MEASURING ANGLES

In order to measure an angle θ, a circular arc centered at the vertex of the angle is drawn, e.g. with a pair of compasses. The length of the arc s is then divided by the radius of the circle r.

The value of θ thus defined is independent of the size of the circle: if the length of the radius is changed then the arc length changes in the same proportion, so the ratio s/r is unaltered. In many geometrical situations, angles that differ by an exact multiple of a full circle are effectively equivalent.

UNITS

Angles are considered dimensionless, since they are defined as the ratio of lengths. There are, however, several units used to measure angles. The *degree* and the *radian* are by far the most common.

With the notable exception of the radian, most units of angular measurement are defined such that one full circle (i.e. one revolution) is equal to n units, for some whole number n. For example, in the case of degrees, $n = 360$.

The degree, denoted by a small superscript circle (°) is 1/360 of a full circle, so one full circle is 360°. One advantage of this old sexagesimal subunit is that many angles common in simple geometry are measured as a whole number of degrees. Fractions of a degree may be written in normal decimal notation (e.g. 3.5° for three and a half degrees), but the following sexagesimal subunits of the "degree-minute-second" system are also in use, especially for geographical coordinates and in astronomy.

The minute of arc is 1/60 of a degree. It is denoted by a single prime (′). For example, 3° 30′ is equal to 3 + 30/60 degrees, or 3.5 degrees. A mixed format with decimal fractions is also sometimes used, e.g. 3° 5.72′ = 3 + 5.72/60 degrees.

The second of arc (or arcsecond, or just second) is 1/60 of a minute of arc and 1/3600 of a degree. It is denoted by a double prime (″). For example, 3° 7′ 30″ is equal to 3 + 7/60 + 30/3600 degrees, or 3.125 degrees.

The radian is the angle subtended by an arc of a circle that has the same length as the circle's radius ($k = 1$ in the formula given earlier). One full circle is 2π radians, and one radian is $180/\pi$ degrees, or about 57.2958 degrees. The radian is abbreviated *rad*, though this symbol is often omitted in mathematical texts, where radians are assumed unless specified otherwise. The radian is used in virtually all mathematical work beyond simple practical geometry, due, for example, to the pleasing and "natural" properties that the trigonometric functions display when their arguments are in radians. The radian is the (derived) unit of angular measurement in the SI system.

The angle of the equilateral triangle is 1/6 of a full circle. It was the unit used by the Babylonians, and is especially easy to construct with ruler and compasses. The degree, minute of arc and second of arc are sexagesimal subunits of the Babylonian unit.

POSITIVE AND NEGATIVE ANGLES

A convention universally adopted in mathematical writing is that angles given a sign are positive angles if measured anticlockwise, and negative angles if measured clockwise, from a given line. If no line is specified, it can be assumed to be the x-axis in the Cartesian plane. In many geometrical situations a negative angle of $-\theta$ is effectively equivalent to a positive angle of "one full rotation less θ". For example, a clockwise rotation of 45° (that is, an angle of −45°) is often effectively equivalent to an anticlockwise rotation of 360° − 45° (that is, an angle of 315°).

IDENTIFYING ANGLES

In mathematical expressions, it is common to use Greek letters (α, β, γ, θ, φ, ...) to serve as variables standing for the size of some angle. Lower case roman letters (a, b, c, ...) are also used. See the figures in this article for examples.

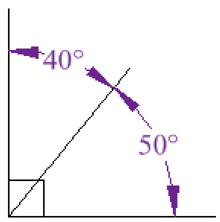
In geometric figures, angles may also be identified by the labels attached to the three points that define them. For example, the angle at vertex A enclosed by the rays AB and AC (i.e. the lines from point A to point B and point A to point C) is denoted

∠BAC or BÂC. Sometimes, where there is no risk of confusion, the angle may be referred to simply by its vertex ("angle A").

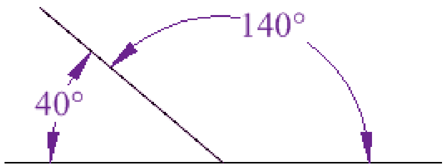
Potentially, an angle denoted, say, ∠BAC might refer to any of four angles: the clockwise angle from B to C, the anticlockwise angle from B to C, the clockwise angle from C to B, or the anticlockwise angle from C to B, where the direction in which the angle is measured determines its sign. However, in many geometrical situations it is obvious from context that the positive angle less than or equal to 180° degrees is meant, and no ambiguity arises. Otherwise, a convention may be adopted so that ∠BAC always refers to the anticlockwise (positive) angle from B to C, and ∠CAB to the anticlockwise (positive) angle from C to B.

<u>TYPES OF ANGLES</u>

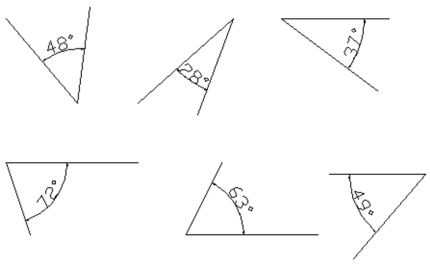
The complementary angles *a (40º)* and *b(50º)* (*b* is the complement of *a*, and *a* is the complement of *b*). They sum to one right angle (90°).



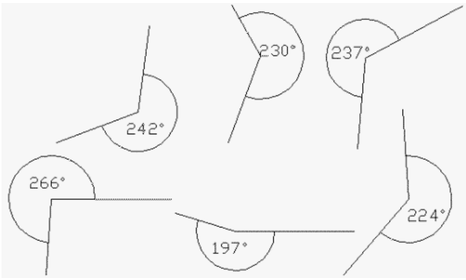
a (40º) and *b(140º)* are supplementary angles because they sum 180º.



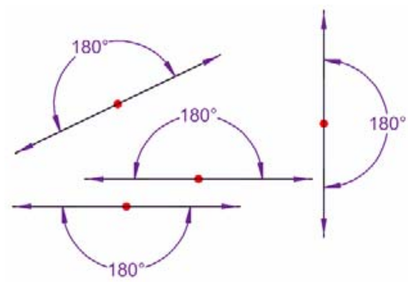
Acute angles. Angles smaller than a right angle (less than 90°) are called acute angles ("acute" meaning "sharp").



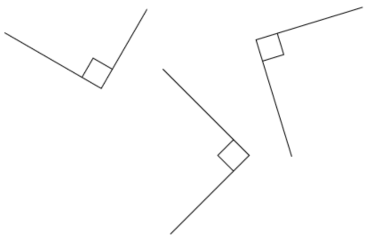
Obtuse angles. Angles larger than a right angle and smaller than two right angles (between 90° and 180°) are called obtuse angles ("obtuse" meaning "blunt").



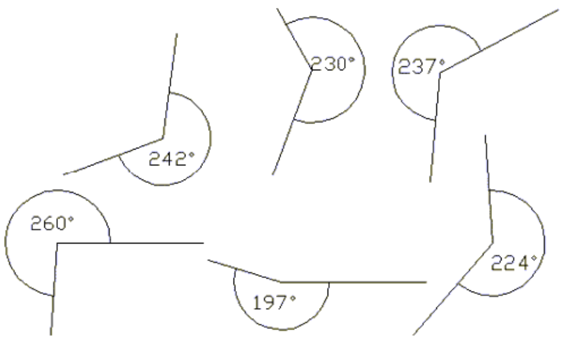
Straight angles Angles equal to two right angles (180°) are called straight angles.



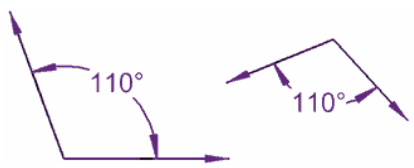
An angle of 90° ($\pi/2$ radians, or one-quarter of the full circle) is called a right angle. Two lines that form a right angle are said to be perpendicular or orthogonal.



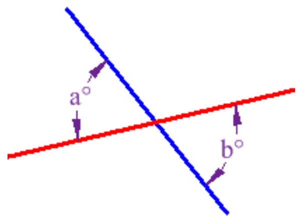
Angles larger than two right angles but less than a full circle (between 180° and 360°) are called reflex angles.



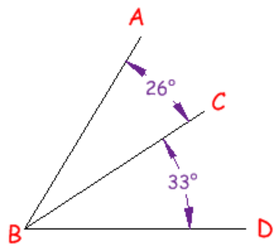
Angles that have the same measure (i.e. the same magnitude) are sometimes said to be congruent, though the diagrams that represent them need not be congruent, so others *(including Euclid)* prefer to say that they are equal in size, or just "equal".



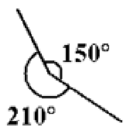
Two angles opposite each other, formed by two intersecting straight lines that form an "X"-like shape, are called vertical angles or opposite angles. These angles are equal in size.



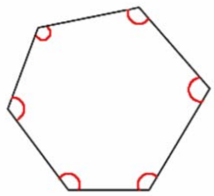
Angles that share a common vertex and edge but do not share any interior points are called adjacent angles.



Two angles that sum to one full circle (360°) are called explementary angles or conjugate angles.



An angle that is part of a simple polygon is called an interior angle if it lies on the inside of that simple polygon. A concave simple polygon has at least one interior angle that exceeds 180°.



In Euclidean geometry, the measures of the interior angles of a triangle add up to π radians, or 180°; the measures of the interior angles of a simple quadrilateral add up to 2π radians, or 360°. In general, the measures of the interior angles of a simple polygon with n sides add up to $[(n - 2) \times \pi]$ radians, or $[(n - 2) \times 180]°$.

The angle supplementary to the interior angle is called the exterior angle. It measures the amount of "turn" one has to make at this vertex to trace out the polygon. If the corresponding interior angle exceeds 180°, the exterior angle should be considered negative. In Euclidean geometry, the sum of the exterior angles of a simple polygon will be 360°, one full turn.

REFERENCE

Wikipedia http://en.wikipedia.org/wiki/Angle